

Math 3305, Chapter 2 Sections 2.4, 2.5 and 2.6 script

Popper 2.4 throughout

Section 2.4 Area

Axioms for this section:

SMSG Axiom 19

Suppose that the region R is the union of two regions R_1 and R_2 . If R_1 and R_2 intersect at most in a finite number of segments and points, then the area of R is the sum of the areas of R_1 and R_2 .

Find the area of this figure: semicircle and a triangle!

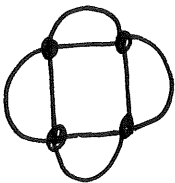


1 segment
2 points

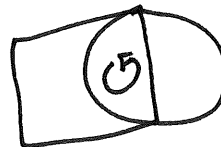


nope

A rounded off square:



4 points
4 segments

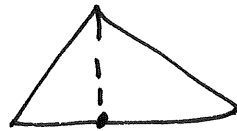


nope

SMSG Axiom 20:

The area of a rectangle is the product of the length of its base and the length of its altitude.

Definition: Altitude



AKA height

Popper 2.4, Question 1

This is the usual formula for area of a rectangle.

- A. True
- B. False

SMSG Axiom 21.

The volume of a rectangular parallelepiped is equal to the product of the length of its altitude and the area of its base.

2.4 Essay Number One

Look up the definition of rectangular parallelepiped and check the formula for the area. Briefly discuss if this is the usual formula. Write the definition in your own words and include a net of the figure (nets: 1.6 pg. 79 textbook).

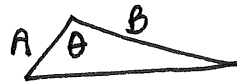
Area of triangle

Standard formula

$$Ax + By + C = 0$$

Trigonometry formula

$$AB \sin \theta$$



Heron's formula



Definition: Semi-Perimeter of a triangle: half the perimeter length. We'll call it "s".

Given a triangle with side lengths a, b, and c. The area of this triangle is

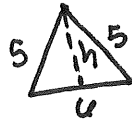
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Let's do two:

An isosceles triangle with sides 5, 5, and 6. $A = \sqrt{s(s-a)(s-b)(s-c)}$

Semiperimeter is $\frac{1}{2}(16) = 8$ $A = \sqrt{8(8-5)(8-5)(8-6)} = (8 \cdot 3 \cdot 3 \cdot 2)^{1/2}$
 $\sqrt{144} = 12$ $\sqrt{96} = 3\sqrt{16} = 12$

And the old way, let's set that up!



get altitude! w/ trig then use formula... but then $h^2 = 5^2 - 3^2$ etc.
 \vdots

And a 3-4-5 right triangle. (scalene!) $A = \sqrt{s(s-a)(s-b)(s-c)}$

Semiperimeter is 6

$$A = \sqrt{6(6-3)(6-4)(6-5)} = (6 \cdot 3 \cdot 2 \cdot 1)^{1/2} = \sqrt{36} = 6$$

$$\frac{1}{2}bh = \frac{1}{2} \cdot 3 \cdot 4 \checkmark \text{ much easier because rt } \Delta$$

Pick's Theorem for the area of any polygon on a geoboard.

(1900 George Pick – the beginning of American contributions to math).

I = the number of interior lattice points

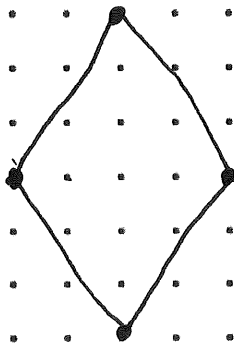
B = the number of lattice points on the ~~interior~~ boundary

$$\text{area} = I + 1/2B - 1$$

Let's look at a rhombus and review



Formula for area of a rhombus:



$$I = 11$$
$$B = 4 \quad \frac{1}{2}B = 2$$

$$11 + 2 - 1 = 12$$

$$P = 6$$

$$Q = 4$$

$$A = 12$$

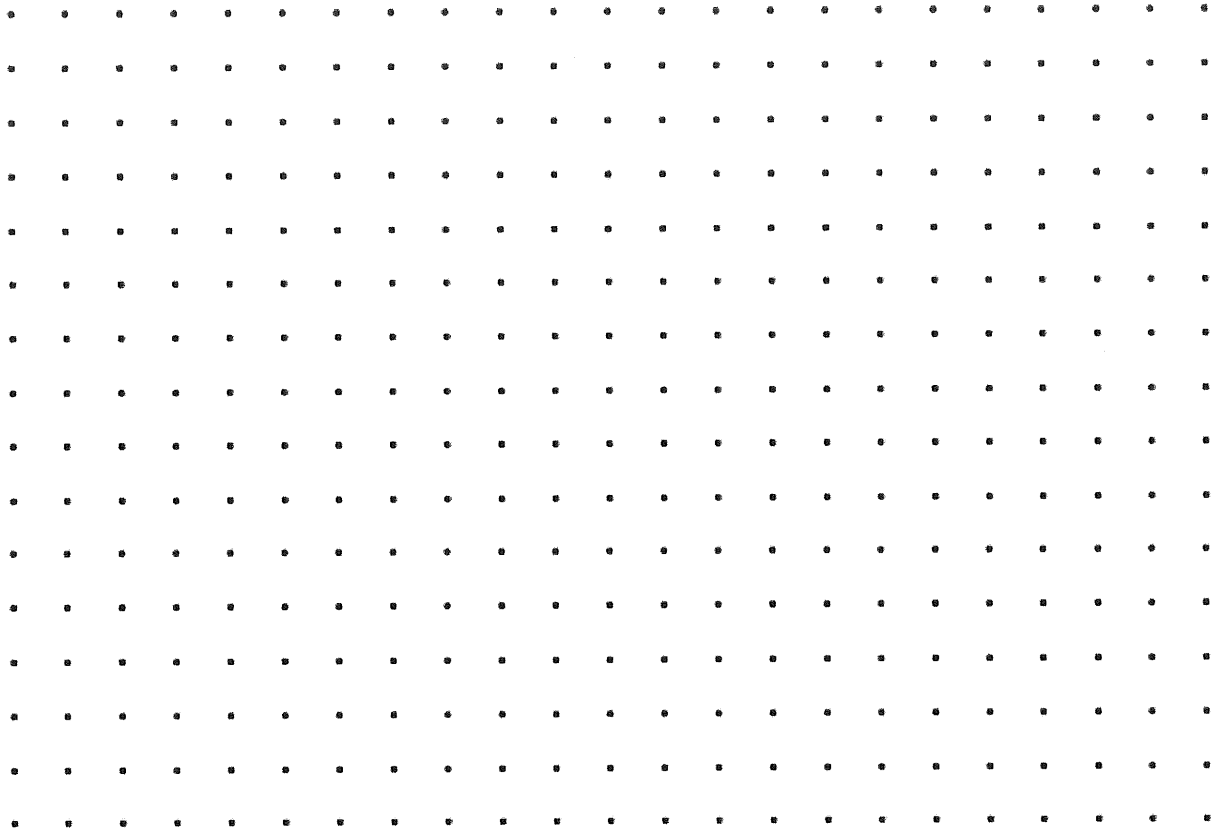
Popper 2.4, Question 2

Which of the following can be used to find the area of a triangle?

- A. Standard formula
- B. Trig formula
- C. Heron's formula
- D. Pick's Theorem
- E. All of the above

2.4 Ms. Leigh's Problem One

Using the grid below, draw a 6 sided CONVEX polygon with at least 2 sides of different lengths. Find it's area using Pick's Theorem. Then write out a way to find the area without Pick's. Which do you prefer! Copy and turn in this page with your homework.

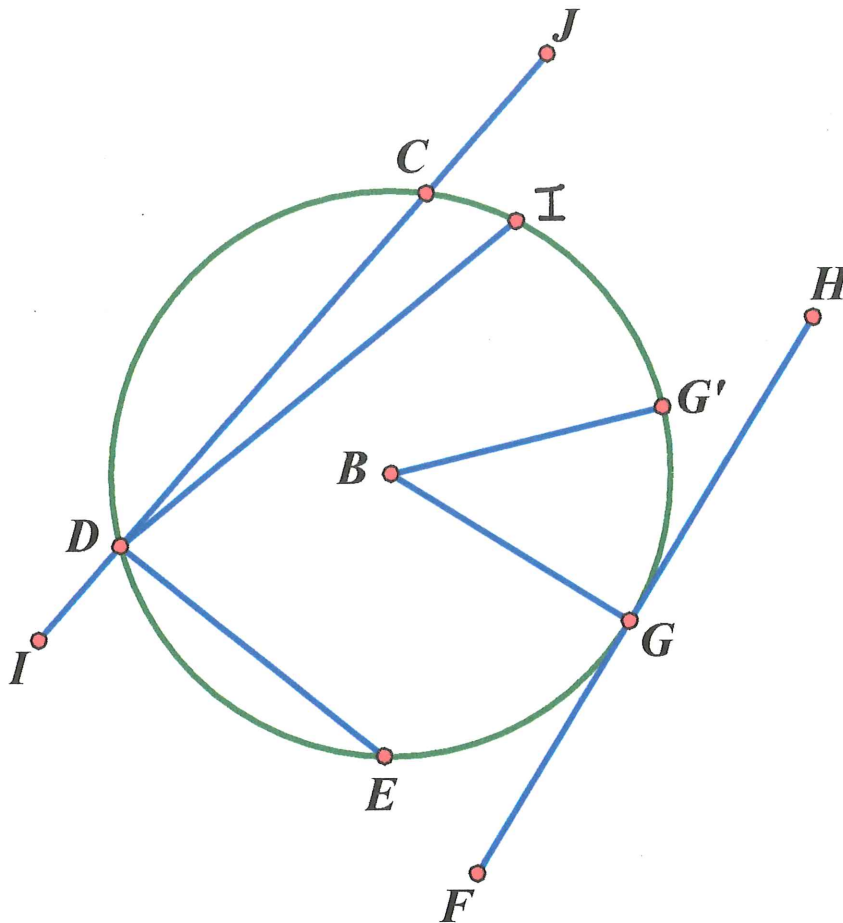


2.5 Circles

Vocabulary looms large in this section.

Fill in the names for the following items

Note that there may be more than one!



Tangent line FH

Secant line JI

Chord DC
 DE
 DI

Radius BG' BG

Central Angle $G'BG$

Inscribed Angle
 DGI
 IDE

Circle points $CIG'GED$

Interior points B

Exterior points $JIFH$

Popper 2.4, Question 3

If you take a secant line and keep the circle points and interior points but discard the exterior points, you will have a chord.

- A. True
- B. False

A quick review: Convert the following to radian measure. Show your steps.

$$180 = \pi \quad \frac{180}{\pi} = 1 \quad \frac{\pi}{180} = 1$$

0 degrees $\cdot 1$ 30 degrees $\frac{\pi}{180} = \frac{\pi}{6}$

$0 \cdot \frac{180}{\pi} = 0$ $0 \cdot \frac{\pi}{180} = 0$

45 degrees $\cdot \frac{\pi}{180} = \frac{\pi}{4}$ 60 degrees $\cdot \frac{\pi}{180} = \frac{\pi}{3}$

90 degrees $\cdot \frac{\pi}{180} = \frac{\pi}{2}$

And now rads to degrees

Pi/6 Pi/3

$$\frac{\pi}{6} \cdot \frac{180}{\pi} = 30 \quad \frac{\pi}{3} \cdot \frac{180}{\pi} = 60$$

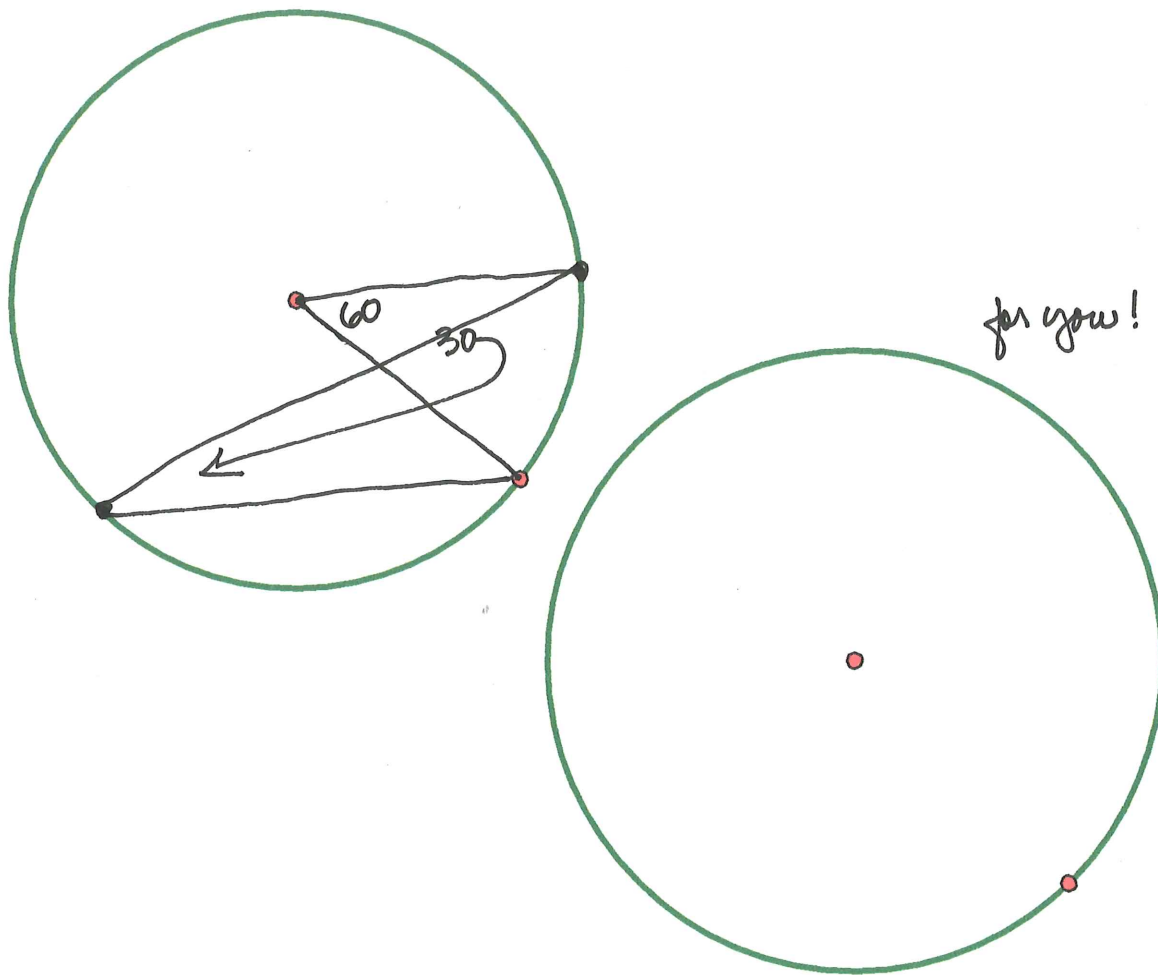
Ms. Leigh's Problem 3

Pretend we can have angles larger than 180 and less than 0. Convert the following to degrees and show your work

- 5pi/4
- 5pi/6
- pi/4

Inscribed angle theorem exercise:

Create two circles with central angles and inscribed angles that SHARE the same endpoints on the circles. Measure both angles. Compare the central angle to the inscribed angle, what did you find?



Popper 2.4, Question 4

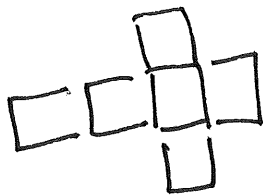
Inscribed angles are made with two radius segments.

- A. True
- B. False

Ms. Leigh's Question 4

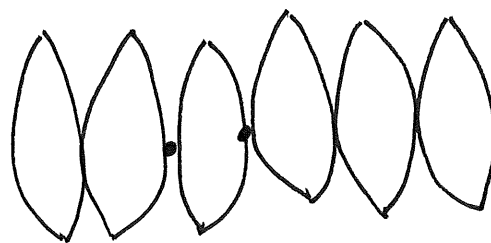
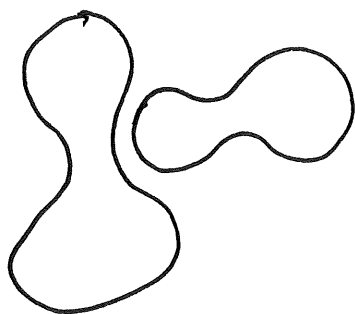
Rewrite Theorem 2.5.1 p. 72 in the textbook in the way that you'd teach it to a 7th grader. Include two illustrations with lines and angles labeled.

We've looked just a bit a nets, now let's bear down a bit. A net is a 2D exploded diagram of a 3D shape. You should be able to cut it out and use tape to create the 3D shape. We've looked at the net of a cube:



And I've assigned you the net of a rectangular parallelepiped.

Let's try a circle net together. I've seen two:



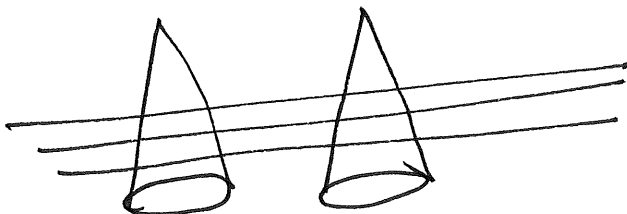
2.4 Ms. Leigh's Problem 5

Look at the nets on p. 78 in the book... they are numbered 18 – 20: When reconstructed as 3D objects what will they look like? Answer the two questions below the illustrations, numbered 21 and 22. Also name figure B.

Moving right along: 2.6 Volumes and Surface Areas

A22. Cavalieri's Principal:

Given two solids and a plane. If for every plane that intersects the solids and is parallel to the given plane, the two intersections determine regions that have the same area, then the two solids have the same volume.



Example:

Volume of related spheres, cylinders, and cones. Given a sphere, a cone, and a right circular cylinder and given that $h = 2r$, let's look at the ratios for their volumes.

Volume of a sphere:

$$\frac{4}{3} \pi r^3$$

Volume of a cone:

$$\frac{1}{3} \pi r^2 h = \frac{2}{3} \pi r^3$$

Volume of a right circular cylinder:

$$\pi r^2 h = 2\pi r^3$$

$$\frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3 = 2\pi r^3$$

cross out πr^3 across

$$\frac{4}{3} = \frac{2}{3} = 2$$

mult. by 3

$$4 = 2 = 6$$

reorder

$$2 = 4 = 6$$

amazing
 (1:2:3)¹⁰

Wrapping up Sections 4, 5, and 6

There's popper 2.4 with 4 questions.

And 2.4 Essay One

2.4 homework #4, #8 and Ms. Leigh's questions 1 and 2

Also the Pick's theorem problem on the dotted array

2.5 homework #4 and Ms. Leigh's questions 3 and 4

2.6 homework #6, #8 and Ms. Leigh's question 5

So next up turn in your homework plus the essays...two different turn ins to different slots in CourseWare. Make sure you've answered all the popper questions. See the Chapter 2 Summary for details and the Course Calendar for deadlines.

Meet you again in Chapter 3.